Scalability Analysis of Grid-Based Multi-Hop Wireless Networks

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***Abstract*—We investigate the scalability of grid-based TDMA wireless networks taking into account protocol details such as routing protocol overheads. Unlike prior work that derives asymptotic scaling laws for general wireless networks, our focus is to characterize the exact relationship between the maximum achievable per node throughput and the network size of grid-based networks under specific protocols. Such a characterization enables for answering questions such as: how many nodes can the network scale to given a traffic demand and what is the impact of control overhead on the effective capacity. While grid-based topologies are highly idealized, the analysis developed in this paper is applicable when such topologies can serve as reasonable approximations for a given network. Further, it also forms a basis for analyzing grid-based random networks that are generalizations of regular grid networks. We present closed-form expressions for achievable rates under both unicast and broadcast traffic for degree 4 and degree 8 regular grid networks. The accuracy of our analytical results is validated using extensive packet-level simulations using the NS-3 simulator.**

***Index Terms*—Capacity, Grid-Based Networks, Scalability, Load Balanced Routing**

I. Introduction

Starting with the seminal work in [1], there has been a lot of work on studying the optimal scaling laws for wireless ad hoc networks. This line of work attempts to characterize the per node throughput capacity as a function of the size of the network. Such characterizations have been studied under different assumptions including assumptions about node capabilities [2], [3], availability of infrastructure support [4], [5], channel models [6], and mobility [7]. Given the vast literature in this area, we refer the interested reader to the excellent surveys [8], [9] for an overview of this work.

Although there has been much work on studying the capacity of wireless ad hoc networks, much of it is based on *asymptotic* analysis. In this setting, typically the number of nodes in the network is allowed to grow to infinity and the goal is to characterize the per node capacity in an order sense. This is done primarily to overcome the technical challenges associated with developing accurate analytical models for random or arbitrary but *finite sized* networks. For example, consider a network of N nodes, each with a fixed communication range R. Suppose the nodes are uniformly and randomly distributed in an area. Then, calculating the exact probability that the network is connected is extremely difficult.

While asymptotic scaling laws provide crucial insights into the growth rate of the throughput capacity with network size, they have limited applicability to finite sized real-world networks [10]. For such networks, having a non-asymptotic model for per node throughput capacity is of much greater use to a network designer. For example, such a model can be useful as a quick tool to answer questions such as how many nodes can a given network scale to given a traffic demand, or what is the impact of control overhead on the effective capacity, that the asymptotic framework cannot address. Indeed, such non-asymptotic models can be very useful as a "back-of-the-envelope" calculation tool even when they are approximate. While general-purpose numerical optimization techniques or simulation based modeling offer alternate means of answering these questions, our interest here is on developing *closed-form* expressions for the performance metrics of interest.

To this end, we focus on grid-based wireless networks in the paper. This simplification allows us to compute expressions for the achievable rates in closed-form by exploiting the symmetry of the network structure. Specifically, we derive closed-form expressions for maximum achievable rates for both unicast and broadcast traffic under different models for routing for degree 4 and degree 8 grid networks. Our analysis explicitly captures of impact of routing and medium access protocols on the effective capacity. We emphasize that while grid-based regular topologies are highly idealized, the analysis developed in this paper is useful when such topologies can serve as reasonable approximation for a given network. Further, it also forms a basis for analyzing grid-based random networks that are generalizations of regular grid networks.

Unlike the asymptotic analyses discussed before, there is relatively little work on computing the exact capacity expressions for wireless networks. This includes recent works [11]–[13] that calculate the exact capacity of mobile ad hoc networks by assuming a simplified *cell-partitioned* model and i.i.d. or Markovian node mobility. These works make use of the 2 hop relaying scheme originally proposed in [7] and rely on message ferrying by mobile nodes. Our focus in this paper is on static grid networks that use multi-hop routing. Static grid networks are also studied in [14] and [15].

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However, unlike these works, we explicitly incorporate the effect of control overheads on the effective capacity. Recently, a framework for non-asymptotic analysis has been described under the banner of "symptotics" [16]. This work seeks to provide approximate non-asymptotic expressions for a variety of topologies, protocols, and traffic models as special cases of a generalized formulation. In contrast, our focus is to study the grid topology in depth, analyzing various ways of forwarding, and computing achievable rates rather than upper bounds.

The rest of the paper is organized as follows. In Sec. II, we present our network model and assumptions. Then we derive closed-form expressions for degree 4 and degree 8 regular grid networks in Sec. III. In Sec. IV, we discuss how the results for regular grid network can be used as a basis for analyzing random grid network. Finally, in Sec. V, we use extensive packet-level simulations using NS-3 to validate the accuracy of our analytical results.

II. Network Model

We consider a wireless network of N identical nodes with a single frequency channel in the network and a single transceiver per node. We assume half duplex operation, so that a node cannot transmit and receive at the same time. We assume that nodes use a *node scheduled* TDMA MAC protocol for channel access. This means that a node's transmission is successful only if no other node within the 2 hop neighborhood of the transmitting node transmits concurrently. Examples of node scheduled TDMA schedules are shown in Figs. 2 and 8.

When a node transmits, the link level transmission rate is given by W bps. All links are assumed to offer identical transmission rates. We study two traffic models: (i) Uniform Unicast, and (ii) Uniform Broadcast. In the unicast model, every node is the source of a unicast session with a destination chosen uniformly at random from the remaining nodes. Each source node sends packets of size B bits at rate λ pps to its destination. In the broadcast model, every node is the source of a broadcast session where the packets generated by the source node must be delivered to all of the other nodes. Each source node generates packets at rate λ pps. The network also uses an underlying routing update and link state protocol. This is discussed in more detail in Sec. III-A3.

Given this model, we are interested in characterizing two quantities: (i) maximum per node throughput, and (ii) maximum feasible network size for a given load per node.

III. Regular Grid Networks

We first consider regular grid networks where every node (except those at the boundaries) has the *same* degree. This simplification allows us to compute expressions for the achievable rates in *closed-form* by exploiting the symmetry of the network structure. In Sec. IV, we build on this analysis to treat grid based random networks.

For illustration, here we focus on grid networks with degrees 4 and 8 (Secs. III-A and III-B respectively). Our approach can be applied to treat regular grid networks with other node degree values as well.

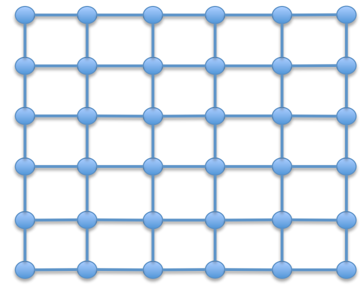

Fig. 1. A Degree 4 Regular Grid Network of 36 nodes.

A. Degree 4 Regular Grid

Consider a regular grid network with degree 4 as shown in Fig. 1. Every internal node in this network has 4 neighboring nodes.[1] In the following, we assume an $N \times N$ grid such that the total number of nodes in the network is given by N^2 where N is an integer.

1) Upper Bounds for Unicast and Broadcast Capacity: We first present a general method to calculate upper bounds on the maximum possible unicast and broadcast traffic rates that this network can support under any MAC and Routing algorithm. For simplicity, we normalize the packet sizes and link bandwidth so that a node can transmit 1 packet per slot.

Consider the uniform unicast traffic model. For any source-destination pair (s, d), let $L_{s,d}$ denote the average path length (in hops) traversed by a packet originating from node s and destined to node d under any routing algorithm. Then the average total number of transmissions per slot required to support N unicast sessions, each generating packets at rate λ pps, is given by

$$\sum_{(s,d)} \lambda L_{s,d}. \tag{1}$$

Let C_{tot} denote the average total number of transmission opportunities per slot resulting from any MAC algorithm subject to the network constraints as discussed in Sec. II. Then, for λ to be feasible, we must have that

$$\sum_{(s,d)} \lambda L_{s,d} \leq C_{tot}. \tag{2}$$

This follows by noting that the total load on the network cannot exceed its transmission capacity. This inequality allows us to calculate an upper bound on the maximum feasible unicast throughput under any routing and scheduling scheme. For example, consider any shortest path routing algorithm. For any source-destination pair (s, d), $L_{s,d}$ is given by the length of a shortest path between s and d. Using the symmetry of the degree 4 regular grid, it can be shown that the average shortest path length $\bar{L}$ between any two nodes satisfies (see [17]):

[1] We do not assume a torus grid to avoid edge effects as in [14], [15].

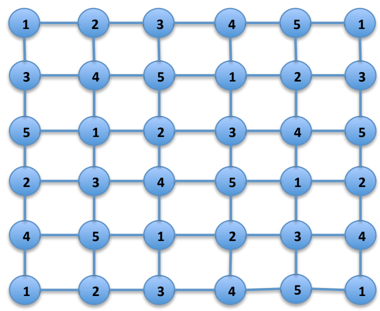

Fig. 2. 5 slot TDMA assignment for the Degree 4 Regular Grid Network

$$\bar{L} \leq \frac{N^3}{N^2 - 1} \tag{3}$$

Further, under the scheduling constraints of the network model in Sec. II, at most $1/2$ of the nodes can transmit concurrently so that $C_{tot} \leq \frac{N^2}{2}$. Using this, we have that any feasible input rate λ must satisfy:

$$\lambda \leq \frac{N^3}{2(N^2 - 1)}. \tag{4}$$

A similar approach can be used to get upper bounds on the maximum feasible broadcast throughput for this network. However, we note that this approach only gives an upper bound that may not be achievable in practice using any routing or scheduling scheme. To compute expressions for the maximum feasible *achievable* rates, in the next section we fix a TDMA schedule and calculate the maximum load that can be supported under three different routing schemes. Our methodology for calculating the maximum feasible rate is based on identifying the bottleneck node (i.e., the node with the most traffic load) under a given routing and scheduling scheme and evaluating the maximum traffic load that it can support.

2) Achievable Rates for Unicast and Broadcast: Consider the node scheduled TDMA assignment for the degree 4 grid as illustrated in Fig. 2. This TDMA schedule consists of a periodic frame with 5 slots per frame. Every node is assigned one slot per frame. The number corresponding to a node in Fig. 2 denotes the slot number in a TDMA frame in which that node is allowed to transmit. It can be seen that this TDMA schedule ensures that a node can transmit only when there is no concurrent transmission in its 2 hop neighborhood. Thus, it is a feasible node scheduled assignment. Note that any two nodes with the same slot number can transmit concurrently and are separated by at least 3 hops. Further, the fraction of time any node can transmit is at most $1/5$ so that the effective link level capacity of any node is $W/5$. A similar "group-based scheduling" model has been considered in [13] to calculate the capacity of a mobile ad hoc network under a cell-partitioned model.

Having fixed the TDMA assignment, we now analyze the achievable rates for uniform unicast traffic under three routing schemes: (i) Balanced Routing, (ii) Non Balanced Routing, and (iii) Random Routing. All of these are variants of shortest path routing. However, they are characterized by different performance bounds. At one extreme, Balanced Routing attempts to distribute the traffic load evenly among all nodes and offers the highest achievable rate. On the other extreme, Non Balanced Routing routes as much traffic as possible through a single node while still selecting shortest path routes, thereby offering the smallest achievable rate. Finally, Random Routing uses a hop-by-hop routing strategy where a node chooses the next hop on a path to the destination as follows. Among all of its neighbors that have the smallest distance to the destination, it chooses one uniformly at random.

To calculate the maximum feasible rate, we note that under each of these three routing schemes, the nodes located at the center of the network receive the highest traffic load under the uniform unicast traffic model.[2] Since all nodes get the same fraction of time to transmit, it is sufficient to focus on the total traffic load on the central nodes for calculating the maximum feasible rate.

Initially, we ignore the protocol overheads such as the Routing and Link state protocol control messages. In Sec. III-A3, we will calculate the "effective" capacity available per node that incorporates the loss in capacity due to these overheads.

Balanced Routing: In Balanced Routing, each source node follows the "Row First, Column Next" based shortest path to route to its destination. This is illustrated in Fig. 3. This strategy ensures that the traffic is evenly distributed among all nodes. In particular, this strategy minimizes the total traffic load on the central node under uniform unicast traffic [15] among all shortest path routing schemes.

To calculate the total expected traffic load on the center node under Balanced Routing, we group the nodes into sets of source and destination nodes such that the traffic generated by the source nodes is relayed by the central node towards the destination nodes. We have 4 cases:

1) Source nodes in upper half, destination nodes on middle lower line.
2) Source nodes in lower half, destination nodes on middle upper line.
3) Source nodes on middle left line, destination nodes in right half.
4) Source nodes on middle right line, destination nodes in left half.

For each of these cases, we can calculate the expected total traffic that passes through the center node. Consider case 1 as an example. Suppose N is odd. Then the total number of source nodes in the upper half is $N(N-1)/2$ while the total number of destination nodes on the middle lower line is $(N-1)/2$. The expected traffic load generated by a source node for a destination node is given by $\lambda/(N^2-1)$.

[2]If N is even, there are four such nodes while if N is odd, there is only one such node.

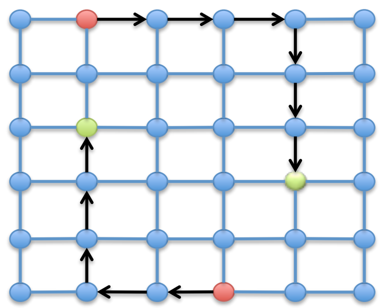

Fig. 3. Illustration of Row First Balanced Routing for the Degree 4 Regular Grid Network

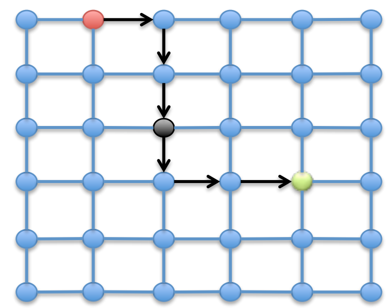

Fig. 4. Illustration of Non Balanced Routing for the Degree 4 Regular Grid Network

Under Balanced Routing, all of this traffic is routed via the center node since the “Row First, Column Next” based shortest path passes through it. Thus, the total expected traffic routed through the center node under case 1 becomes:

$$\frac{N(N-1)}{2} \times \frac{(N-1)}{2} \times \frac{\lambda}{(N^2-1)} \tag{5}$$

The total expected traffic routed through the center node under the other cases can be similarly calculated. Finally, the center node generates traffic of its own at rate λ. Summing these together, it can be shown that the total expected traffic load on the center node is given by:

$$\frac{N^3-2N}{N^2-1}\lambda \tag{6}$$

Since this cannot exceed the transmission capacity $W/5$ of the center node, we have

$$\lambda \leq \frac{N^2-1}{N^3-2N} \times \frac{W}{5} \tag{7}$$

Non Balanced Routing: In Non Balanced Routing, each source node follows the “Row First, Middle Column Next, Row Last” based shortest path to route to its destination. This is illustrated in Fig. 4. This strategy distributes the traffic in an uneven manner among all nodes. In particular, among all shortest path routing schemes, this strategy maximizes the total traffic load on the central node under uniform unicast traffic.

To calculate the total expected traffic load on the center node under Non Balanced Routing, we group the nodes into sets of source and destination nodes such that the traffic generated by the source nodes is relayed by the central node towards the destination nodes. We have 4 cases:

1) Source nodes in upper left quadrant, destination nodes in lower right quadrant.
2) Source nodes in upper right quadrant, destination nodes in lower left quadrant.
3) Source nodes in lower left quadrant, destination nodes in upper right quadrant.
4) Source nodes in lower right quadrant, destination nodes upper left quadrant.

For each of these cases, we can calculate the expected total traffic that passes through the center node. Consider case 1 as an example. Suppose N is even. Then the total number of source nodes in the upper left quadrant is $(N-2)^2/4$ while the total number of destination nodes in lower right quadrant is $N^2/4$. The expected traffic load generated by a source node for a destination node is given by $\lambda/(N^2-1)$. Under Non Balanced Routing, all of this traffic is routed via the center node since the “Row First, Middle Column Next, Row Last” based shortest path passes through it. Thus, the total expected traffic routed through the center node under case 1 becomes:

$$\frac{(N-2)^2}{4} \times \frac{N^2}{4} \times \frac{\lambda}{(N^2-1)} \tag{8}$$

The total expected traffic routed through the center node under the other cases can be similarly calculated. Finally, the center node generates traffic of its own at rate λ. Summing these together, it can be shown that the total expected traffic load on the center node is given by:

$$\frac{N^4+3N^3-N^2-12N+8}{4(N^2-1)}\lambda \tag{9}$$

Since this cannot exceed the transmission capacity $W/5$ of the center node, we have

$$\lambda \leq \frac{4(N^2-1)}{N^4+3N^3-N^2-12N+8} \times \frac{W}{5} \tag{10}$$

Random Routing: Next, we consider Random Routing which uses the following hop-by-hop routing strategy. Here, a node chooses the next hop on a path to the destination as follows. Among all of its neighbors that have the smallest distance to the destination, it chooses one uniformly at random. This is illustrated in Fig. 5. This strategy can be thought of as performing a random walk while staying on shortest paths and always moving strictly closer to the destination in each hop.

To calculate the total expected traffic load on the center node under Random Routing, similar to the analysis for Non Balanced Routing, we group the nodes into sets of source and destination nodes such that the traffic generated by the source nodes is likely to be relayed by the central node towards the destination nodes. We have 4 cases:

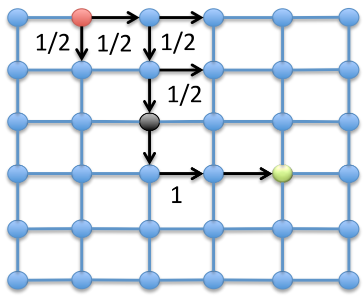

Fig. 5. Illustration of Random Routing for the Degree 4 Regular Grid Network

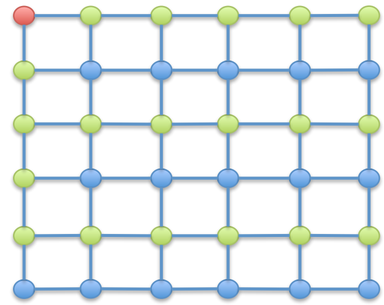

Fig. 6. Illustration of Every Alternate Row Broadcast for the Degree 4 Regular Grid Network

1) Source nodes in upper left quadrant, destination nodes in lower right quadrant.
2) Source nodes in upper right quadrant, destination nodes in lower left quadrant.
3) Source nodes in lower left quadrant, destination nodes in upper right quadrant.
4) Source nodes in lower right quadrant, destination nodes upper left quadrant.

For each of these cases, we can calculate the expected total traffic that passes through the center node. Consider case 1 as an example. Suppose N is odd. Because of the hop-by-hop nature of this strategy, a source node in the upper left quadrant has the *same* probability of routing via the center node for all destinations in the lower right quadrant. Further, the sum of these probabilities for all source nodes at the same distance from the center node is upper bounded by 1. Summing over all nodes, the sum total probability is upper bounded by $(N-1)/2$. The total number of destination nodes in lower right quadrant is $(N-1)^2/4$. Thus, the expected traffic load that is routed via the center node is upper bounded by

$$\frac{(N-1)}{2} \times \frac{(N-1)^2}{4} \times \frac{\lambda}{(N^2-1)} \tag{11}$$

The total expected traffic routed through the center node under the other cases can be similarly calculated. Finally, the center node generates traffic of its own at rate λ. Summing these together, it can be shown that the total expected traffic load on the center node is upper bounded by

$$\frac{\phi}{(N^2-1)}\lambda \text{ where } \phi = 1.5(N-1)^2\left(\frac{N-5}{2} + 2^{\frac{3-N}{2}}\right) + 2N^2 - 2N + 8(N-1)\left(\frac{N^2-17}{16} + 2^{\frac{1-N}{2}}\right) + 8\left(1 - 2^{\frac{3-N}{2}}\right) \tag{12}$$

Since this cannot exceed the transmission capacity $W/5$ of the center node, we have

$$\lambda \leq \frac{(N^2-1)}{\phi} \times \frac{W}{5} \tag{13}$$

Next, we analyze the achievable rate for uniform broadcast traffic. We assume that the broadcast is performed according to the "Every Alternate Row Broadcast" strategy as shown in Fig. 6. We note that under broadcast traffic, there is no single bottleneck node. Rather, every node seems the same traffic load. Thus, to calculate the maximum achievable rate under any strategy, it suffices to calculate the total number of broadcast transmissions per packet. For the "Every Alternate Row Broadcast" strategy as shown in Fig. 6, this is given by $N^2/2 + N/2 - 1$. Thus, the maximum feasible broadcast traffic rate in given by:

$$\lambda \leq \frac{2}{N^2+N-2} \times \frac{W}{5} \tag{14}$$

3) Incorporating Control Overheads: So far, we have ignored the overheads associated with control messages that are used by Routing and Link Layer protocols. These control messages must be sent along with the regular data packets and end up reducing the effective capacity available to a node. Let RO and LO denote the average per node overhead associated with any Routing and Link Layer protocol. Then the effective bandwidth W_{eff} available to transmit data packets is given by:

$$W_{\text{eff}} = W - RO - LO \tag{15}$$

We first calculate an expression for RO under typical link state routing protocols (such as OLSR). Suppose on average every node generates link state updates (LSUs) of size C every T_R seconds. These LSUs are broadcast to all the remaining nodes. Assuming the "Every Alternate Row Broadcast" strategy, it follows that the average routing overhead per node (in packets/slot or pps) is given by:

$$RO = \left(\frac{N^2}{2} - \frac{N}{2} - 1\right) \times \frac{C}{BT_R} \tag{16}$$

We next calculate an expression for LO under typical link layer protocols. Suppose on average every node generates link layer HELLO updates of size H every T_L seconds. These HELLOs are broadcast to all the neighboring nodes. It follows that the average link layer overhead per node (in pps) is given by:

Traffic Model	Routing Model	Throughput per node (pps)
Unicast	Balanced	$\lambda \leq \frac{N^2-1}{N^3-2N} \times W_{\text{eff}}$
Unicast	Non Balanced	$\lambda \leq \frac{4(N^2-1)}{N^4+3N^3-N^2-12N+8} \times W_{\text{eff}}$
Unicast	Random	$\lambda \leq \frac{(N^2-1)}{\phi} \times W_{\text{eff}}$ (ϕ is given by (12))
Broadcast	Every Alternate Row	$\lambda \leq \frac{2}{N^2+N-2} \times W_{\text{eff}}$

TABLE I
SUMMARY OF RESULTS FOR DEGREE 4 REGULAR GRID.

$$LO = \frac{H}{BT_L} \tag{17}$$

Using these, the effective bandwidth W_{eff} (in pps) available to transmit data packets is given by:

$$W_{\text{eff}} = \frac{W}{5B} - \left(\frac{N^2}{2} - \frac{N}{2} - 1\right) \times \frac{C}{BT_R} - \frac{H}{BT_L} \tag{18}$$

Table I summarizes the results for degree 4 regular grid. It can be seen from the expressions that the per node unicast throughput under balanced routing scales as $O(1/\sqrt{M})$ where M is the size of the network. This agrees with the scaling order obtained in [1]. On the other hand, the per node unicast throughput under non balanced routing scales as $O(1/M)$. Finally, it is interesting to note that the per node unicast throughput under random routing also scales as $O(1/\sqrt{M})$. Thus, it is off from the optimal routing performance only by a constant factor.

B. Degree 8 Regular Grid

Consider a regular grid network with degree 8 as shown in Fig. 7. Every internal node in this network has 8 neighboring nodes. In the following, we assume an $N \times N$ grid such that the total number of nodes in the network is given by N^2 where N is an integer.

1) Upper Bounds for Unicast and Broadcast: Similar to Sec. III-A1, we can derive upper bounds on the maximum possible unicast traffic rate that this network can support under any MAC and Routing algorithm. Consider any shortest path routing algorithm. For any source-destination pair (s, d), $L_{s,d}$ is given by the length of a shortest path between s and d. Using the symmetry of the degree 8 regular grid, it can be shown that the average shortest path length $\bar{L}$ between any two nodes satisfies:

$$\bar{L} \leq \frac{4N^3 + 11N^2 + 14N2}{6(N+2)(N^21)} \tag{19}$$

Further, under the scheduling constraints of the network model in Sec. II, at most $1/4$ of the nodes can transmit concurrently so that $C_{tot} \leq \frac{N^2}{4}$. Using this, we have that any feasible input rate λ must satisfy:

$$\lambda \leq \frac{1.5(N+2)(N^21)}{4N^3 + 11N^2 + 14N - 2} \tag{20}$$

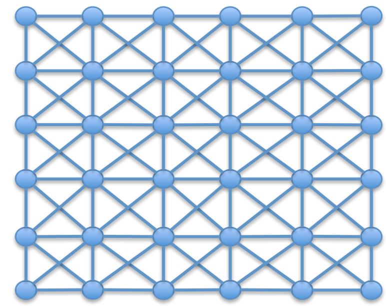

Fig. 7. Degree 8 Regular Grid Network

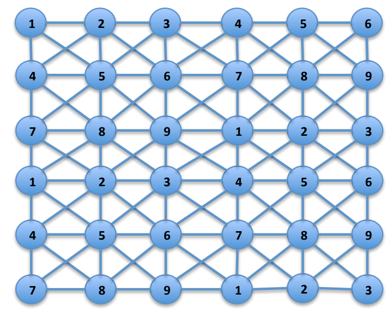

Fig. 8. 9 slot TDMA assignment for the Degree 8 Regular Grid Network

A similar approach can be used to get upper bounds on the maximum feasible broadcast throughput for this network. However, as before, these upper bound are not necessarily achievable in practice using any routing or scheduling scheme. To compute expressions for the maximum feasible *achievable* rates, in the next section we fix a TDMA schedule and calculate the maximum load that can be supported under three different routing schemes. As before, our methodology for calculating the maximum feasible rate is based on identifying the bottleneck node (i.e., the node with the most traffic load) under a given routing and scheduling scheme.

2) Achievable Rates for Unicast and Broadcast: Consider the node scheduled TDMA assignment for the degree 8 grid as illustrated in Fig. 8. This TDMA schedule consists of a periodic frame with 9 slots per frame. Every node is assigned one slot per frame. The number corresponding to a node in Fig. 8 denotes the slot number in a TDMA frame in which that node is allowed to transmit. It can be seen that this TDMA schedule ensures that a node can transmit only when there is no concurrent transmission in its 2 hop neighborhood. Thus, it is a feasible node scheduled assignment. Note that any two nodes with the same slot number can transmit concurrently and are separated by at least 3 hops. Further, the fraction of time any node can transmit is at most $1/9$.

Similar to the degree 4 grid case, we analyze the achievable unicast rates for this network under three routing schemes: (i) Balanced Routing, (ii) Non Balanced Routing, and (iii) Random Routing. As before, under each of these three routing schemes, the nodes located at the center of the network carry

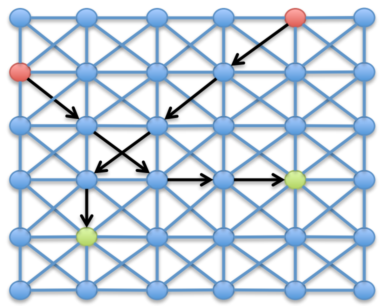

Fig. 9. Illustration of Diagonal First Balanced Routing for the Degree 8 Regular Grid Network

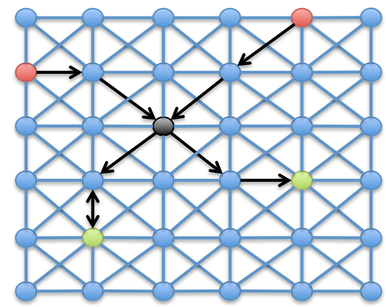

Fig. 10. Illustration of Non Balanced Routing for the Degree 8 Regular Grid Network

the highest traffic load under the uniform unicast model.[3] Since all nodes get the same fraction of time to transmit, it is sufficient to focus on the total traffic load on the central nodes for calculating the maximum feasible rate.

Balanced Routing: In Balanced Routing, each source node follows the "Diagonal First, Row/Column Next" based shortest path to route to its destination. This is illustrated in Fig. 3. This strategy ensures that the traffic is evenly distributed among all nodes. In particular, this strategy minimizes the total traffic load on the central node under uniform unicast traffic.

Similar to the analysis for Balanced Routing for the degree 4 grid, we calculate the total expected traffic load on the center node by appropriately grouping nodes into sets of source and destination nodes such that the traffic generated by the source nodes is relayed by the central node towards the destination nodes. For each of these cases, we then calculate the expected total traffic that passes through the center node. Finally, the center node generates traffic of its own at rate λ. Summing these together, it can be shown that the total expected traffic load on the center node is given by:

$$\frac{7N^3 - 9N^2 + 3N + 2}{4(N^2 - 1)}\lambda \tag{21}$$

Since this cannot exceed the transmission capacity $W/9$ of the center node, we have

$$\lambda \leq \frac{4(N^2 - 1)}{7N^3 - 9N^2 + 3N + 2} \times \frac{W}{9} \tag{22}$$

Non Balanced Routing: In Non Balanced Routing, each source node chooses a shortest path to its destination that passes through the center node. This is illustrated in Fig. 10. This strategy distributes the traffic in an uneven manner among all nodes. In particular, among all shortest path routing schemes, this strategy maximizes the total traffic load on the central node under uniform unicast traffic.

Similar to the analysis for Non Balanced Routing for the degree 4 grid, we calculate the total expected traffic load on the center node by appropriately grouping nodes into sets of source and destination nodes such that the traffic generated by the source nodes is relayed by the central node towards the destination nodes. For each of these cases, we then calculate the expected total traffic that passes through the center node. Finally, the center node generates traffic of its own at rate λ. Summing these together, it can be shown that the total expected traffic load on the center node is given by:

[3] If N is even, there are four such nodes while if N is odd, there is only one such node.

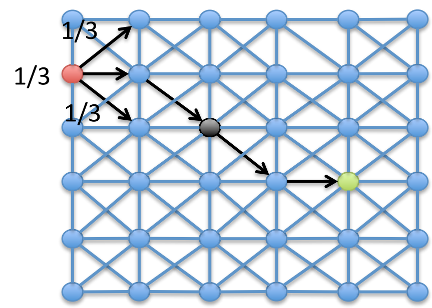

Fig. 11. Illustration of Random Routing for the Degree 8 Regular Grid Network

$$\frac{\frac{N^4}{8} + N^3 + 6N - 3}{N^2 - 1}\lambda \tag{23}$$

Since this cannot exceed the transmission capacity $W/9$ of the center node, we have

$$\lambda \leq \frac{N^2 - 1}{\frac{N^4}{8} + N^3 + 6N - 3} \times \frac{W}{9} \tag{24}$$

Random Routing: Finally, we consider Random Routing which uses a hop-by-hop routing strategy. Here, a node chooses the next hop on a path to the destination as follows. Among all of its neighbors that have the smallest distance to the destination, it chooses one uniformly at random. This is illustrated in Fig. 11.

To calculate the total expected traffic load on the center node under Random Routing, we follow a procedure similar to the analysis for Random Routing over degree 4 grid. For brevity, we only present the main result. It can be shown that the total expected traffic load on the center node is upper bounded by

Traffic Model	Routing Model	Throughput per node (pps)
Unicast	Balanced	$\lambda \leq \frac{4(N^2-1)}{7N^3-9N^2+3N+2} \times W_{\text{eff}}$
Unicast	Non Balanced	$\lambda \leq \frac{N^2-1}{\frac{N^4}{8}+N^3+6N-3} \times W_{\text{eff}}$
Unicast	Random	$\lambda \leq \frac{(N^2-1)}{\psi} \times W_{\text{eff}}$ (ψ is given by (25))
Broadcast	Every Alternate Row	$\lambda \leq \frac{2}{N^2+N-2} \times W_{\text{eff}}$

TABLE II
SUMMARY OF RESULTS FOR DEGREE 8 REGULAR GRID.

$$\frac{\psi}{(N^2-1)}\lambda \text{ where } \psi = 2(N-1)^3 + 5(N-1)^2 + (N^2-1)(2-2^{\frac{3-N}{2}}) \qquad (25)$$

Since this cannot exceed the transmission capacity $W/9$ of the center node, we have

$$\lambda \leq \frac{(N^2-1)}{\psi} \times \frac{W}{9} \qquad (26)$$

Next, we analyze the achievable rate for uniform broadcast traffic. We assume that the broadcast is performed according to the "Every Alternate Row Broadcast" strategy as shown in Fig. 12. We note that under broadcast traffic, there is no single bottleneck node. Rather, every node seems the same traffic load. Thus, to calculate the maximum achievable rate under any strategy, it suffices to calculate the total number of broadcast transmissions per packet. For the "Every Alternate Row Broadcast" strategy as shown in Fig. 12, this is given by $N^2/2 + N/2 - 1$. Thus, the maximum feasible broadcast traffic rate in given by:

$$\lambda \leq \frac{2}{N^2+N-2} \times \frac{W}{5} \qquad (27)$$

Finally, the effective bandwidth W_{eff} (in pps) available to transmit data packets is given by:

$$W_{\text{eff}} = \frac{W}{9B} - (\frac{N^2}{2} - \frac{N}{2} - 1) \times \frac{C}{BT_R} - \frac{H}{BT_L} \qquad (28)$$

Table II summarizes the results for degree 8 regular grid. Similar to the degree 4 case, we find that the per node unicast throughput under both balanced routing and random routing scales as $O(1/\sqrt{M})$ where M is the size of the network.

IV. GRID BASED RANDOM NETWORKS

In this section, we show how the analysis presented earlier for regular grid networks can be used as a basis to analyze more general networks. Specifically, we consider a class of random networks that we call "Grid Based Random Networks". We assume that a network of this class is formed as follows. First, we partition the network into disjoint "cells" and assume that there is at least one node per cell. We assume that nodes in adjacent cells ate within communication range of each other. These two assumptions ensure that a network in this class is always a connected network. The remaining nodes are located uniformly at random over the cells. This is illustrated in Fig. 13 where is is assumed that nodes in a cell can communicate with nodes in same cell and adjacent cells to the Left, Right, Top, and Bottom.

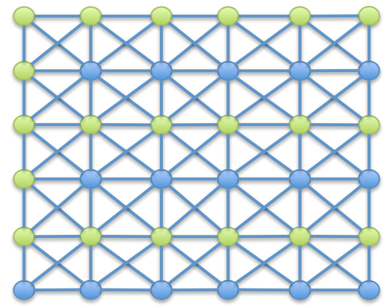

Fig. 12. Illustration of Every Alternate Row Broadcast for the Degree 8 Regular Grid Network

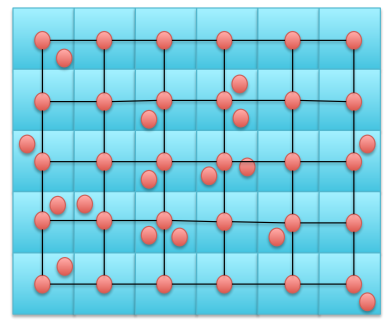

Fig. 13. Illustration of the Degree 4 Random Grid Network Model.

We assume there can me at most one concurrent transmission per cell and its adjacent cells. Define the node density d as the ratio of the total number of nodes to the number of cells. Note that $d \geq 1$ since we assume at least one node per cell. The degree of node density in this network can be controlled by varying this parameter d.

We note that this network is equivalent to a Regular Grid network where each "cell" can be thought of as a node with traffic intensity proportional to the number of nodes in that cell. Thus, we can apply the analysis for Regular grid on this equivalent network to derive expressions for maximum feasible unicast and broadcast throughput.

V. SIMULATION BASED VALIDATION

In this section, we use NS-3 simulations [18] to validate the theoretical analysis of the previous sections. The objective is to understand how well the modeling framework approximates the performance as seen in packet-level simulations. We will also try to understand and explain any discrepancies between the results obtained from the theoretical analysis and the results obtained via simulation.

To validate these results using simulation we make two approximations.

- The theoretical analysis assumes uniform all-pair unicast traffic: each node uniformly randomly chooses a destina-

	4-Degree Grid	8-Degree Grid
Network structure		
Distance between nodes	1 km	1 km
Transmission range of a node	1 km	1.415 km
TDMA Parameters		
Data rate	5 Mbps	5 Mbps
Slot time	3400 μs	5000 μs
Guard time	100 μs	100 μs
Interframe time	0 μs	0 μs
Packet queue size	20,000 packets	20,000 packets
OLSR Parameters		
Hello update frequency	2 s	2 s
Link state routing update frequency	5 s	5 s

TABLE III

NETWORK STRUCTURE, TDMA, AND OLSR PARAMETERS USED IN THE SIMULATIONS. AFTER 10 SECONDS OF NO ACTIVITY, A PACKET IS CONSIDERED TO BE LOST. INTERFERENCE BETWEEN NODES IS NOT MODELED. THE SLOT TIME IS LONGER FOR THE 8-DEGREE GRID THAN FOR THE 4-DEGREE GRID TO ACCOMMODATE THE LARGER CONTROL PACKETS. TO ADDRESS THE ISSUE OF LOST CONTROL PACKETS FOR THE 8-DEGREE GRID WE ADDITIONALLY PUSHED CONTROL PACKETS IN AT THE FRONT OF THE TDMA QUEUE RATHER THAN AT THE BACK.

tion for each packet to transmit. In the simulations we use uniform random flows: for a given simulation run, each node sends packets to one randomly chosen destination for the duration of the simulation, rather than choosing a different destination for each packet.

- The theoretical analysis considers balanced, non balanced, and random shortest path routing algorithms. In the simulations we use OLSR as the routing algorithm.

Using these approximations we compare the saturation throughput obtained from the simulations with the throughput results given by the theoretical analysis.

A. Saturation Throughput

1) Definition: Saturation throughput is typically defined as the throughput achieved when the network is overloaded (i.e., the transmission queues are never empty). Since we are interested in the throughput that can be achieved without dropping packets, we slightly modify this definition and instead define saturation throughput as the lowest pps for which queue lengths become unstable over the life of the simulation.

We observe in our simulation results that queue lengths experience a sharp jump up once the average queue length per node reaches 0.5 packets. We thus identify the pps at which saturation throughput occurs in our simulations by determining the smallest pps for which the average queue length per node is 0.5 packets.

B. NS-3 Simulation Setup

1) Network Models:

- *4-Degree grid.* We use the network structure, TDMA, and OLSR parameters given in Table V-A1. To schedule node transmissions, we use the 5-slot assignment shown in Figure 2.
- *8-Degree Grid* We use the network structure, TDMA, and OLSR parameters given in Table V-A1. To schedule node transmissions, we use the 9-slot assignment shown in Figure 8.

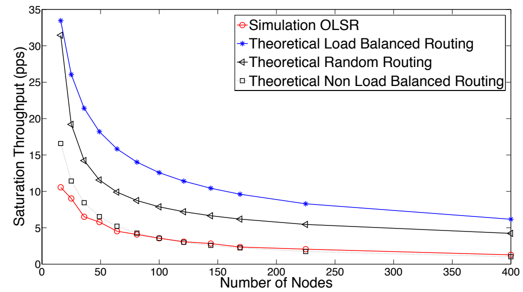

Fig. 14. Saturation Throughput for degree 4 grid.

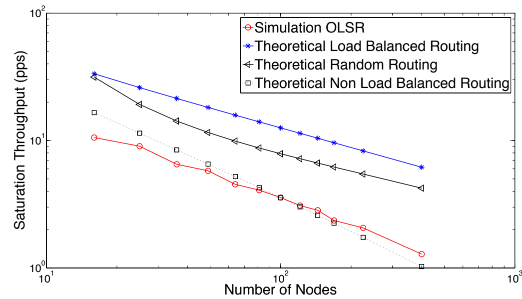

Fig. 15. Saturation Throughput vs. Network Size for degree 4 grid (log-log scale).

2) Network Traffic: We use 1000 byte data packets and unicast UDP flows where each node randomly selects a destination and sends packets to that destination for the duration of a simulation run. All flows were started at time $t = 50$ s + jitter, and stopped at $t = 100$ s. The simulation time for each run is 100 s, and we perform 50 simulation runs, selecting different random destinations on each run.

C. NS-3 Simulation Results

Figures 14 and 16 plot the theoretical results in Section III for $N = 16$ to 400 for degree 4 and 8 grid networks respectively, along with the saturation throughput obtained from the NS-3 simulations. We use the methodology described before to determine this value.

In the degree 4 case, we observe that as N increases, the saturation throughput appears to converge to the theoretical non-load-balanced shortest path throughput, indicating that the performance of OLSR based routing is far from the load balanced case. However, note that the shortest path selection mechanism of OLSR should be closest to the random routing strategy since the theoretical non-load-balanced case deliberately routes all flows via the center node. To investigate this further, in Figure 15 we plot Figure 14 in the log-log scale. This allows us to compare the *scaling* performance of OLSR with the three schemes. From Figure 15, it can be seen that the slope of the OLSR curve is closest to the random routing case. Thus, the performance gap between OLSR and random routing

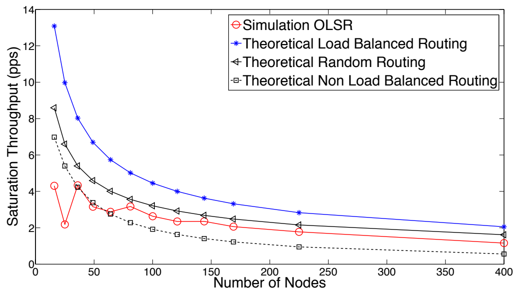

Fig. 16. Saturation Throughput for degree 8 grid.

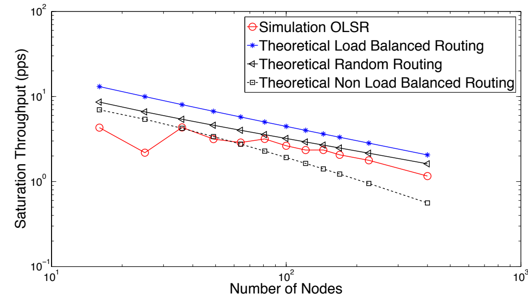

Fig. 17. Saturation Throughput vs. Network Size for degree 8 grid (log-log scale).

in Figure 14 appears to be a contant factor term (independent of network size) that may be due to other protocol overheads and/or approximations made in the theoretical analysis.

In the degree 8 case (Figure 16), we observe that as N increases, the saturation throughput appears to converge values between the theoretical non-load-balanced and random routing throughput, again indicating that the performance of OLSR based routing is far from the load balanced case. We also plot this in the log-log scale in Figure 17. It can again be seen that the slope of the OLSR curve is closest to the random routing case. Thus, the performance gap between OLSR and random routing in Figure 14 appears to be a contant factor term (independent of network size) that may be due to other protocol overheads and/or approximations made in the theoretical analysis.

Finally, we observe that the saturation throughput is lower for the 8-degree grid as compared with the 4-degree grid. This may be due to more shortest paths using the same diagonal links and consequently, the bottleneck nodes having higher contention in the 8-degree grid than in the 4-degree grid.

VI. Conclusions

In this paper, we presented non-asymptotic analysis of per node throughput for grid-based wireless networks. This approach differs from the majority of work on deriving asymptotic capacity scaling laws and is more suitable as a design guideline for real-world networks. We presented closed-form expressions for achievable rates under both unicast and broadcast traffic for degree 4 and degree 8 regular grid networks. The accuracy of our analytical results was validated using extensive packet-level simulations using NS-3.

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References

[1] P. Gupta and P. R. Kumar. The Capacity of Wireless Networks. *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388-404, Mar. 2000.

[2] S. Aeron and V. Saligrama. Wireless Ad Koc Networks: Strategies and Scaling Laws for the Fixed SNR Regime. *IEEE Trans. Inf. Theory*, vol. 53, no. 6, pp. 2044-2059, Jun. 2007.

[3] A. Ozgur, O. Leveque, and D. Tse. Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad Hoc Networks. *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3549-3572, Oct. 2007.

[4] B. Liu, P. Thiran, and D. Towsley. Capacity of a Wireless Ad Hoc Network with Infrastructure. In *Proc. ACM MobiHoc*, Sep. 2007.

[5] U. Kozat and L. Tassiulas. Throughput Capacity of Random Ad Hoc Networks with Infrastructure Support. In *Proc. ACM MobiCom*, Jun. 2003. v

[6] S. Li, Y. Liu, and X. Li. Capacity of Large Scale Wireless Networks Under Gaussian Channel Model. In *Proc. ACM MobiCom*, 2008.

[7] M. Grossglauser and D. Tse. Mobility Increases the Capacity of Ad Hoc Wireless Networks. *IEEE/ACM Trans. Netw.*, vol. 10, no. 4, pp.477-486, Aug. 2002.

[8] F. Xue and P. R. Kumar. Scaling Laws for Ad-Hoc Wireless Networks: An Information Theoretic Approach. *Foundations and Trends in Networking*, vol. 1, no. 2, 2006.

[9] A. Ozgur, O. Leveque, and D. Tse. Operating Regimes of Large Wireless Networks. *Foundations and Trends in Networking*, vol. 5, no. 1, 2009.

[10] R. Ramanathan, R. Allan, P. Basu, J. Feinberg, G. Jakllari, V. Kawadia, S. Loos, J. Redi, C. Santivanez and J. Freebersyser. Scalability of Mobile Ad Hoc Networks: Theory vs Practice. In *Proc. MILCOM* , Oct. 2010.

[11] M. J. Neely and E. Modiano. Capacity and Delay Tradeoffs for Ad-Hoc Mobile Networks. *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 1917-1937, June 2005.

[12] R. Urgaonkar and M. J. Neely. Network Capacity Region and Minimum Energy Function for a Delay-Tolerant Mobile Ad Hoc Network. *IEEE/ACM Trans. Networking*, vol. 19, no. 4, pp. 1137-1150, August 2011.

[13] J. Liu, X. Jiang, H. Nishiyama, and N. Kato. Exact Throughput Capacity under Power Control in Mobile Ad Hoc Networks. In *Proc. IEEE INFOCOM*, 2012.

[14] G. Mergen and L. Tong. Stability and Capacity of Regular Wireless Networks. *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 1938-1953, June 2005.

[15] G. Barrenetxea, B. Berefull-Lozano, and M. Vetterli. Lattice Networks: Capacity Limits, Optimal Routing, and Queueing Behavior. *IEEE/ACM Trans. Networking*, vol. 14, no. 3, pp. 492-505, June 2006.

[16] R. Ramanathan, A. Samanta, T. La Porta. Symptotics: A Framework for Analyzing Real-World Wireless Networks. In *Proc. ACM PE-WASUN*, Cyprus, October 2012.

[17] J. Silvester and L. Kleinrock. On the Capacity of Multihop Slotted ALOHA Networks with Regular Structure. *IEEE Transactions on Communications*, vol. 31, no. 8, pp. 974-982, Aug. 1983.

[18] NS-3 Simulator, http://www.nsnam.org/